Recovered equations and ordinates of the Göttingen 765 airfoil

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Abstract The Göttingen 765 is known as the root-section airfoil of the Me163 Komet, a rocket-powered tailless interceptor of WWII fame. But no definitive ordinates of the Göttingen 765 exist. While surviving drawings and museum examples provide low-resolution data on the shape of the airfoil, it is often useful to have more exact equations of the ordinates for research purposes. Here we derive a meanline and thickness distribution that fit these measurements.

Keywords: airfoils, Me163, Göttingen 765

Table 1. Symbols

a_0, a_1, a_2, a_3	Coefficients of forebody thickness distribution
d_0, d_1, d_2, d_3	Coefficients of afterbody thickness distribution
h	Principal component coefficient
I	Leading edge sharpness parameter
m	Position of maximum thickness as a fraction of chord
M_n	Nominal meanline
M_e	Empirical meanline
r_{LE}	Leading edge radius
T	Airfoil thickness as a fraction of chord
x	A position along airfoil chord
x_u	Upper surface x ordinate
x_l	lower surface x ordinate
y	A distance above or below airfoil chord
y_l	lower surface y ordinate
y_t	thickness distribution at given x
y_u	upper surface y ordinate
y_P	Meanline principal component
y_S	Meanline sigmoid component
y_R	Meanline residual component
λ	Sigmoid component coefficient
θ	Angle of meanline slope

1 The Meanline

For tailless aircraft, the shape of the meanline is particularly important because such aircraft require a wing that has a coefficient of moment that is both close to zero, and stable through a wide range of angles of attack. The papers of Alexander Lippisch, chief designer of the Messerschmidt Me163, are at Iowa State University in Ames. Among them is a graph showing the camber line of the Göttingen 765 and four other similar airfoils, differing by the amount of reflex at the trailing edge [1]. Although there are no equations given for camber lines on the drawing, [2] states that the meanline for the Göttingen 765 is the sum of these two equations:

$$y_p = h(1 - (1 - 2x)^{2n}) \tag{1}$$

$$y_s = \lambda((1 - 2x) - (1 - 2x)^{2m+1}) \tag{2}$$

where m and n are integers. Although the values of h, λ, m, and n are not given, it is possible to recover the values of those coefficients using multiple regression against the ISU curve. A very close fit occurs where $m = 2$, $n = 2$, $h = 0.00573952678$, and $\lambda = 0.018521412$. But if Lippisch were working with only a slide rule, he could not have carried more than two or "two-and-a-half" significant digits into any calculation. Rounding to that precision gives us $h = .00575$ and $\lambda = .0185$. The nominal meanline then becomes

$$M_n = .00575(1 - (1 - 2x)^4) + .0185((1 - 2x) - (1 - 2x)^5) \tag{3}$$

and differentiating gives

$$\frac{dy}{dx} = .046(-8x^3 + 12x^2 - 6x + 1) + .148(20x^4 - 40x^3 + 30x^2 - 10x + 1) \tag{4}$$

Equations 3 and 4 can be used to generate a nominal airfoil by wrapping the standard NACA 4-digit thickness distribution around the meanline of [2], given the known thickness of the airfoil (14.4% of chord). Since the NACA distribution was originally derived by reference to the Göttingen 398 airfoil [3], it stands to reason that the thickness distribution of the 765 should be at least somewhat similar to NACA's standard. That thickness distribution is given by the well-known equation:

$$\pm\, y_t = \frac{T}{.2}(.2969\sqrt{x} - .126x - .3516x^2 + .2846x^3 - .1015x^4) \tag{5}$$

For any given position on the chord ($0 \leq x \leq 1$) ordinates of the upper and lower surface can be generated as follows: y_t values for the given x are found from equation 5; the meanline m_n for x is found from equation 3, and its slope is found from equation 4. The slope angle θ is simply $\theta = \arctan(dy/dx)$. Then ordinates for the upper surface are $x_u = x - y_t \sin\theta$; $y_u = m_n + y_t \cos\theta$; and for the lower surface are $x_l = x + y_t \sin\theta$; $y_l = m_n - y_t \cos\theta$.

This computed airfoil can be compared to the actual Göttingen 765. Measured ordinates for the airfoil are found at [1], in several versions. The highest quality and best-measured version is one made from a Messerschmitt factory drawing, which I will take as standard in this paper. Figure 1 is a foreshortened graph showing this profile against the profile just computed.

It is clear from figure 1 that the computed airfoil, though vaguely similar to the Göttingen 765, has a large flaw in the middle section, where the computed profile shows less camber than the actual airfoil. This can only be due to an error in the meanline of [2], implying that both the ISU curve and [2] are not fully correct in their treatment of the meanline. It may well be that the originally intended meanline was modified after wind tunnel testing showed a need for increased camber. If that is the case, it is possible that surviving documentation reflects the original meanline rather than the one used in the actual aircraft.

To investigate this further, I found y-ordinates for the computed airfoil corresponding to the x-ordinates for each point in the drawing dataset [1]. By subtracting the computed airfoil y-ordinates from the y-ordinates of the drawing, a set of residual values can be obtained. These residuals are shown in figure 2.

The mid-chord flaw shows up clearly as a large hump in this figure. Note also that, if our thickness distribution were incorrect it would show up in figure 2 as a gap between the residuals of the upper and lower surfaces. In both cases, those gaps are generally small except at the leading and trailing edges.

Notice too that at the leading edge, the drawing dataset has significantly smaller ordinates (less apparent camber) than the computed airfoil. This is almost certainly an artifact of measurement, since

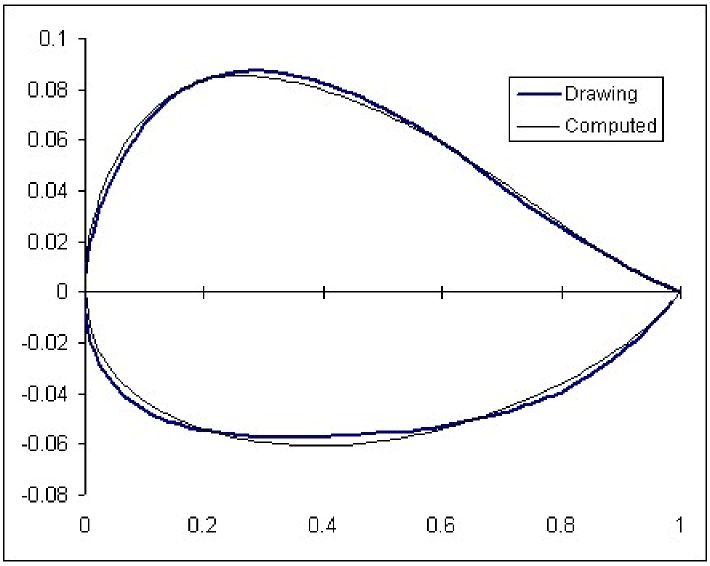

Figure 1. Foreshortened plot of the Messerschmitt factory drawing compared to computed profile. Errors are clearly present.

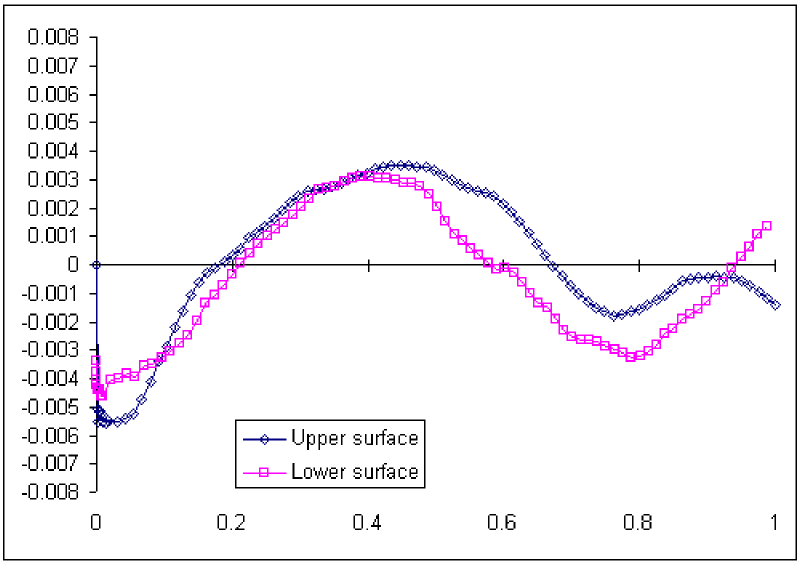

Figure 2. Residuals of the y-ordinates, using the meanline of [2] and NACA-4 digit thickness.

the exact point of an airfoil's leading edge chord is often difficult to determine by eye. To correct this, we rotate the ordinates of the drawing data [1] around the trailing edge clockwise by .0048 radian, so the residuals near the leading edge come close to zero. Re-drawing the residuals after this rotation gives us a residual dataset that is nearly zero at both leading and trailing edges (figure 3), which is what we want.

The reason for the rotation is that we would like to flatten that hump. To do that, we need to add a new constituent equation to the meanline that will reduce these residuals to reasonably low values. But any meanline must cross the axis at $x = 0$ and $x = 1$, which means the only way to correct residuals at those points is by rotation. Those few datapoints at the very leading edge, which show great disparities from the computed airfoil, are an artifact of the way the drawing was measured; since the profile is nearly vertical at the leading edge, slight errors in measuring the x-ordinate result in spuriously large errors in the y-ordinate. These residuals can be safely ignored. Again we use multiple regression to find an equation

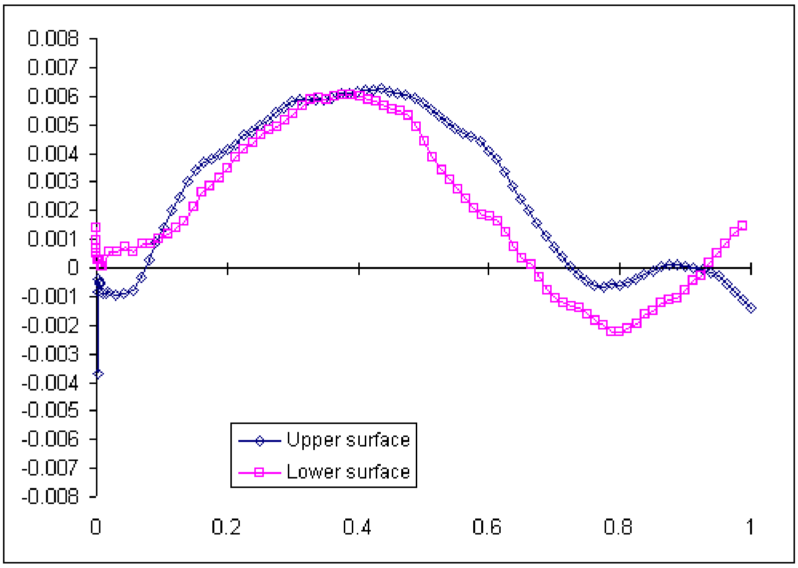

Figure 3. Same as figure 2, with drawing rotated .0048 radian clockwise around trailing edge.

that fits the residual data closely, constraining coefficients so that $y = 0$ at $x = 1$, $x = 0$. In this case rounding to sliderule precision is not adequate for the task, but using slightly more precision we find:

$$y_R = -0.4148x^5 + 1.1607x^4 - 1.0954x^3 + 0.3705x^2 + .021x \tag{6}$$

Combining equations 3 and 6 gives us the empirical meanline:

$$M_e = .00575(1-(1-2x)^4)+.0185((1-2x)-(1-2x)^5)-0.4148x^5+1.1607x^4-1.0954x^3+0.3705x^2-.021x \tag{7}$$

and differentiating gives

$$\frac{dy}{dx} = .046(-8x^3+12x^2-6x+1)+0.148(20x^4-40x^3+30x^2-10x+1)-2.074x^4+4.6428x^3-3.2862x^2+0.741x-.021 \tag{8}$$

This meanline has a maximum camber of .019 at 26% of chord. Following the same procedure as before, we wrap the NACA 4-digit thickness distribution around the empirical meanline and derive a "first cut" empirical version of the Göttingen 765. Comparing this to figure 3 shows that we have indeed corrected the camber.

2 The thickness distribution

Plotting the residuals of the empirical airfoil to the drawing dataset shows problem areas in a small slice near the leading edge, and a broader region of the trailing edge. In both cases, the empirical airfoil is too thick, causing the upper-surface and lower-surface residuals to diverge.

The standard NACA 4-digit distribution has a small gap at the trailing edge, amounting to about 1% of the airfoil thickness. It is clear from the residual plot that this gap is responsible for the large residual at the trailing edge. Therefore the thickness distribution must be modified. A procedure for generating modified thickness distributions can be found in [4]. In addition to the trailing edge gap, there are three other parameters that can be varied by this procedure: the trailing edge angle; the leading edge radius; and m, the position of maximum thickness. Specifying the trailing edge gap and slope provides two of the four coefficients we need for the thickness distribution.

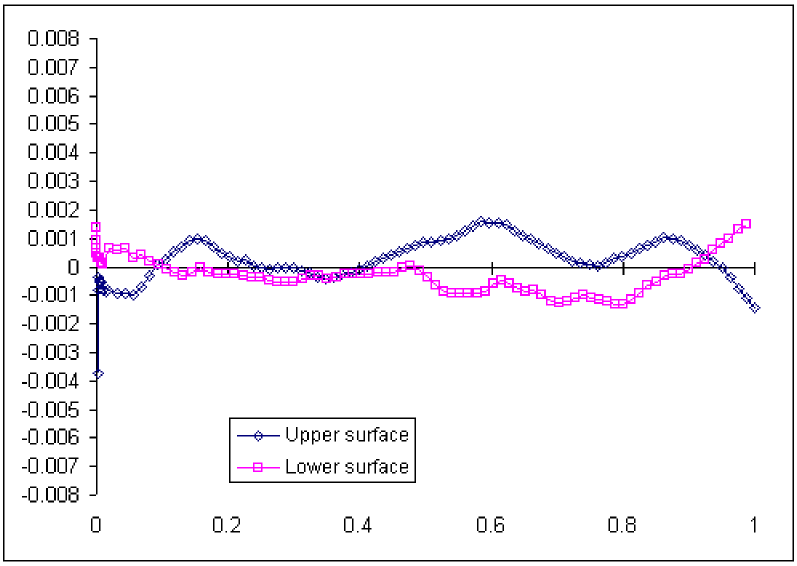

Figure 4. Residuals after adding empirical correction to the meanline of Ref [2].

$$d_0 = \frac{\textit{Trailing edge gap}}{2} \tag{9}$$

For our purposes, we set $d_0 = 0$ to close the trailing edge gap entirely. Next,

$$d_1 = \textit{Trailing edge slope} \tag{10}$$

Reference [4] provides a table with suggested values for the trailing edge slope based on the position of maximum thickness $m = x$, chosen to avoid a reflexive surface. We start by assuming the standard value of $m = .3$ for the Göttingen 765.

Table 2.

m	d_1
.2	1.000 T
.3	1.170 T
.4	1.575 T
.5	2.235 T
.6	3.500 T

In Table 2, T is thickness expressed as a fraction of the chord. If other positions of m are desired, the figures in this table are closely replicated by

$$d_1 = (15.83333m^3 - 2.17857m^2 - .240476m + 1.009)T \tag{11}$$

Since maximum thickness of the Göttingen 765 is at $x = .3$, we provisionally adopt $d_1 = 1.170T = .16848$. Then we compute

$$d_2 = \frac{3[T - 2d_0 - 2d_1(1-m)]}{2(1-m)^2} + \frac{d_1}{(1-m)} \tag{12}$$

and

$$d_3 = \frac{-d_1}{3(1-m)^2} - \frac{2d_2}{3(1-m)} \tag{13}$$

These four parameters determine the thickness distribution aft of the point of maximum thickness. Forward of that point, we define an index parameter I that controls the leading edge radius and the thickness of the nose. In the standard case $I = 6$ but this can be modified within the range $0 \leq I < 9$. Brief experimentation shows that $I = 5.7$ gives a nearly perfect fit to the nose of the airfoil (and is much superior to the standard NACA 4-digit distribution).

Then, following [5], we compute the following:

$$a_0 = .296904\frac{I}{6} \tag{14}$$

$$p = \frac{0.2(1-m)^2}{.588 - 2d_1(1-m)} \tag{15}$$

$$a_1 = \left(\frac{0.3}{m}\right) - \left(\frac{15a_0}{8\sqrt{m}}\right) - \left(\frac{m}{10p}\right) \tag{16}$$

$$a_2 = -\left(\frac{0.3}{m^2}\right) + \left(\frac{5a_0}{4m^{3/2}}\right) + \left(\frac{1}{5p}\right) \tag{17}$$

$$a_3 = \left(\frac{0.1}{m^3}\right) - \left(\frac{.375a_0}{m^{5/2}}\right) - \left(\frac{1}{10pm}\right) \tag{18}$$

The leading edge radius is given by

$$r_{LE} = \frac{1}{2}\left(\frac{T}{0.2}a_0\right)^2 \tag{19}$$

Finally, the thickness distribution ahead of the maximum thickness point $x = m$ is found by

$$\pm\, y_t = \frac{T}{.2}(a_0\sqrt{x} + a_1x + a_2x^2 + a_3x^3) \tag{20}$$

and aft of the maximum thickness point by

$$\pm\, y_t = d_0 + d_1(1-x) + d_2(1-x)^2 + d_3(1-x)^3 \tag{21}$$

Since this is different from the standard NACA 4-digit distribution, we can expect the residual plot to change too; and the changes are particularly beneficial to the leading and trailing edges. In fact, the fit is so close that there is a good possibility that the NACA modified distribution was in fact the distribution used in the Göttingen 765. If true, this may be the only known case of the NACA modified 4-digit thickness distribution being used in a German aircraft of WWII.

Looking at the final residual plot (figure 5), both surfaces of our recovered airfoil are within .001 of the Messerschmitt drawing for most of the length. The leading edge area is particularly well fit. Only the trailing edge continues to show a discrepancy, from 75% of the chord back, where the Messerschmitt drawing is slightly thicker than our recovered version. This may be due to slight modifications needed for control surfaces. It is possible to remove that trailing edge discrepancy by increasing the trailing edge slope parameter d_1, but that creates a bigger problem that it solves farther forward, when the recovered version becomes noticeably thicker than the drawn version.

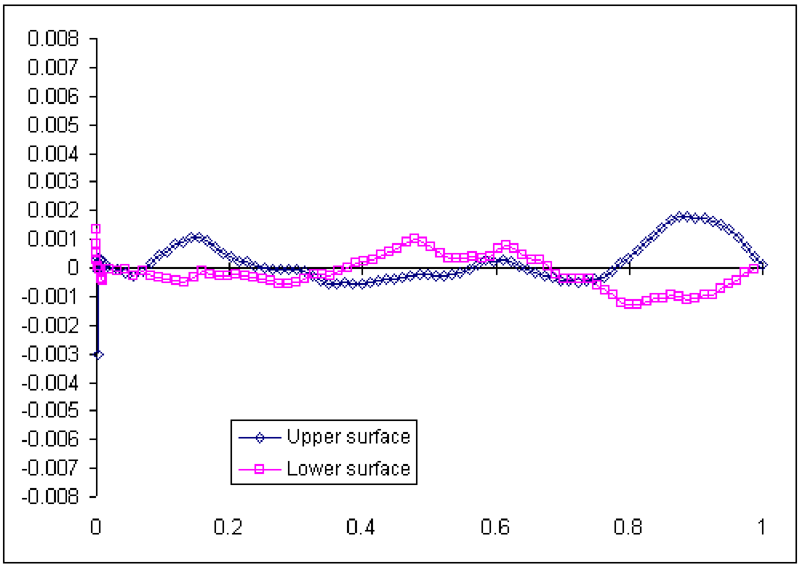

Figure 5. Residuals after applying modified thickness distribution to the empirical meanline.

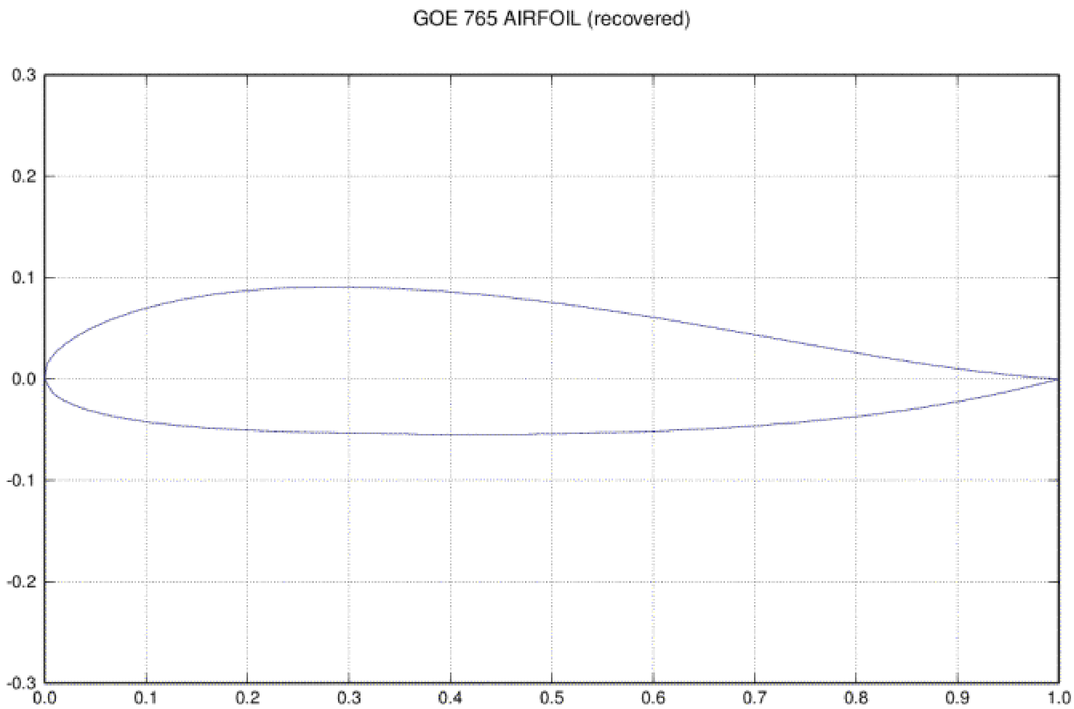

Figure 6. Scale drawing of final recovered airfoil.

3 Appendix A

Ordinates for the recovered Göttingen 765 airfoil.

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Göttingen 765 airfoil (recovered). Thick: .144 LE radius: 0.02062 LE slope: 0.17300
1.000000 -0.000000
0.989894 0.000681
0.979818 0.001451
0.969768 0.002306
0.959744 0.003241
0.949742 0.004251
0.939760 0.005332
0.929797 0.006480
0.919852 0.007690
0.909921 0.008959
0.900004 0.010282
0.890099 0.011656
0.880204 0.013078
0.870319 0.014543
0.860440 0.016049
0.850569 0.017592
0.840702 0.019170
0.830839 0.020778
0.820980 0.022415
0.811122 0.024076
0.801265 0.025760
0.791409 0.027463
0.781551 0.029184
0.771693 0.030918
0.761832 0.032664
0.751969 0.034419
0.742102 0.036181
0.732232 0.037948
0.722357 0.039717
0.712477 0.041485
0.702591 0.043251
0.692701 0.045013
0.682804 0.046768
0.672900 0.048514
0.662990 0.050250
0.653073 0.051973
0.643149 0.053681
0.633218 0.055373
0.623278 0.057046
0.613331 0.058699
0.603376 0.060330
0.593413 0.061938
0.583441 0.063519
0.573461 0.065074
0.563472 0.066599
0.553475 0.068093
0.543469 0.069555
0.533454 0.070983
0.523430 0.072376
0.513397 0.073731
0.503355 0.075047
0.493303 0.076322
0.483243 0.077555
0.473172 0.078745
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0.463093	0.079889
0.453003	0.080986
0.442904	0.082034
0.432796	0.083032
0.422677	0.083979
0.412549	0.084871
0.402410	0.085709
0.392262	0.086490
0.382103	0.087212
0.371934	0.087874
0.361755	0.088474
0.351565	0.089011
0.341365	0.089482
0.331155	0.089885
0.320934	0.090220
0.310703	0.090484
0.300461	0.090675
0.290208	0.090785
0.279946	0.090803
0.269673	0.090725
0.259390	0.090547
0.249098	0.090261
0.238797	0.089864
0.228488	0.089348
0.218172	0.088708
0.207849	0.087937
0.197522	0.087028
0.187191	0.085974
0.176858	0.084767
0.166525	0.083397
0.156195	0.081855
0.145870	0.080131
0.135554	0.078212
0.125250	0.076086
0.114964	0.073738
0.104700	0.071149
0.094465	0.068300
0.084267	0.065166
0.074117	0.061714
0.064025	0.057907
0.054009	0.053690
0.044090	0.048989
0.039176	0.046425
0.034298	0.043693
0.029464	0.040768
0.024683	0.037614
0.019965	0.034182
0.015330	0.030393
0.010806	0.026119
0.006448	0.021104
0.002392	0.014696
-0.000000	0.000000
0.007608	-0.012992
0.013552	-0.017745
0.019194	-0.021153
0.024670	-0.023870
0.030035	-0.026149
0.035317	-0.028118
0.040536	-0.029854

0.045702	-0.031406
0.050824	-0.032810
0.055910	-0.034090
0.065991	-0.036349
0.075975	-0.038289
0.085883	-0.039981
0.095733	-0.041470
0.105535	-0.042792
0.115300	-0.043972
0.125036	-0.045030
0.134750	-0.045983
0.144446	-0.046843
0.154130	-0.047620
0.163805	-0.048323
0.173475	-0.048960
0.183142	-0.049537
0.192809	-0.050060
0.202478	-0.050534
0.212151	-0.050962
0.221828	-0.051349
0.231512	-0.051699
0.241203	-0.052014
0.250902	-0.052297
0.260610	-0.052551
0.270327	-0.052779
0.280054	-0.052981
0.289792	-0.053162
0.299539	-0.053322
0.309297	-0.053469
0.319066	-0.053609
0.328845	-0.053740
0.338635	-0.053862
0.348435	-0.053973
0.358245	-0.054075
0.368066	-0.054164
0.377897	-0.054242
0.387738	-0.054306
0.397590	-0.054358
0.407451	-0.054395
0.417323	-0.054417
0.427204	-0.054424
0.437096	-0.054414
0.446997	-0.054388
0.456907	-0.054345
0.466828	-0.054283
0.476757	-0.054203
0.486697	-0.054104
0.496645	-0.053984
0.506603	-0.053844
0.516570	-0.053683
0.526546	-0.053500
0.536531	-0.053295
0.546525	-0.053066
0.556528	-0.052813
0.566539	-0.052535
0.576559	-0.052232
0.586587	-0.051903
0.596624	-0.051546
0.606669	-0.051161

0.616722	-0.050748
0.626782	-0.050305
0.636851	-0.049831
0.646927	-0.049325
0.657010	-0.048787
0.667100	-0.048215
0.677196	-0.047607
0.687299	-0.046964
0.697409	-0.046284
0.707523	-0.045565
0.717643	-0.044806
0.727768	-0.044006
0.737898	-0.043163
0.748031	-0.042276
0.758168	-0.041343
0.768307	-0.040363
0.778449	-0.039333
0.788591	-0.038253
0.798735	-0.037121
0.808878	-0.035934
0.819020	-0.034690
0.829161	-0.033388
0.839298	-0.032026
0.849431	-0.030600
0.859560	-0.029110
0.869681	-0.027552
0.879796	-0.025925
0.889901	-0.024225
0.899996	-0.022451
0.910079	-0.020600
0.920148	-0.018668
0.930203	-0.016654
0.940240	-0.014555
0.950258	-0.012367
0.960256	-0.010088
0.970232	-0.007715
0.980182	-0.005244
0.990106	-0.002674
1.000000	-0.000000

4 Appendix B

Polars for the recovered Göttingen 765 airfoil. Data generated with XFOIL, $n_{\text{crit}} = 9$.

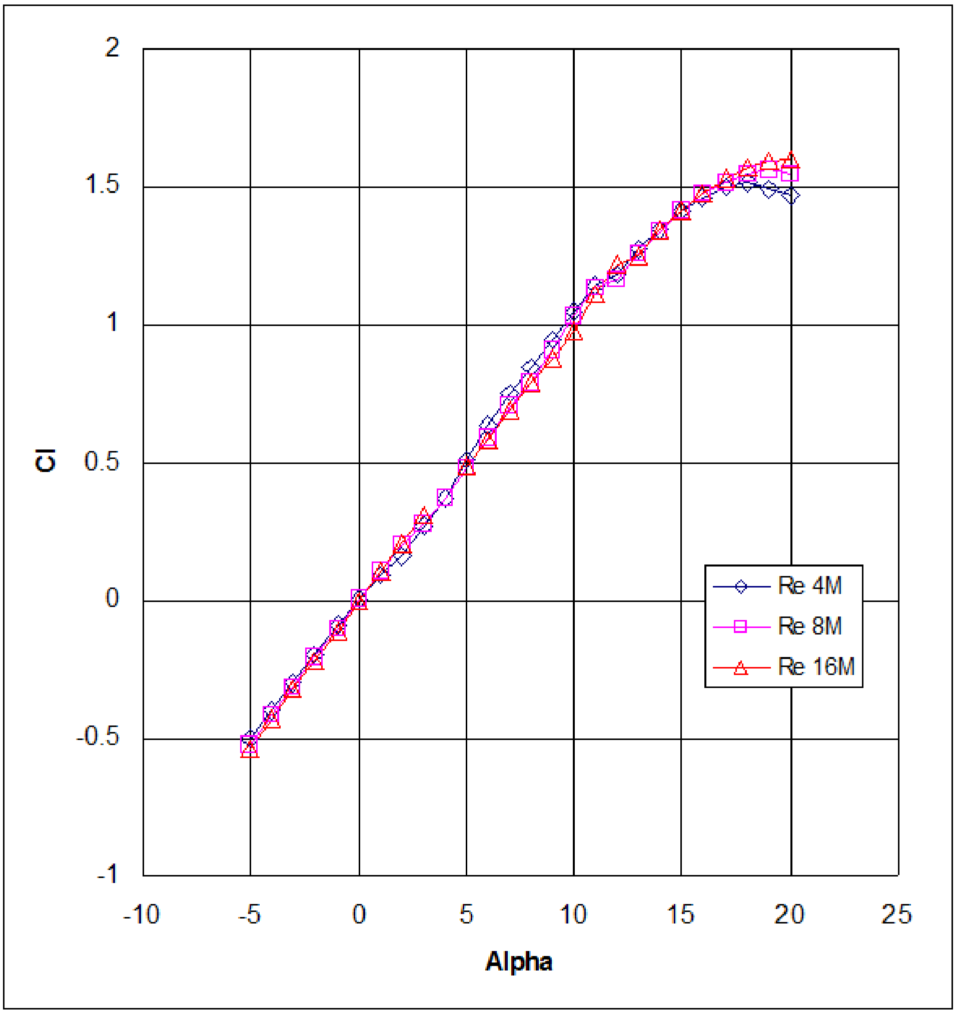

Figure 7. Angle of attack vs. coefficient of lift.

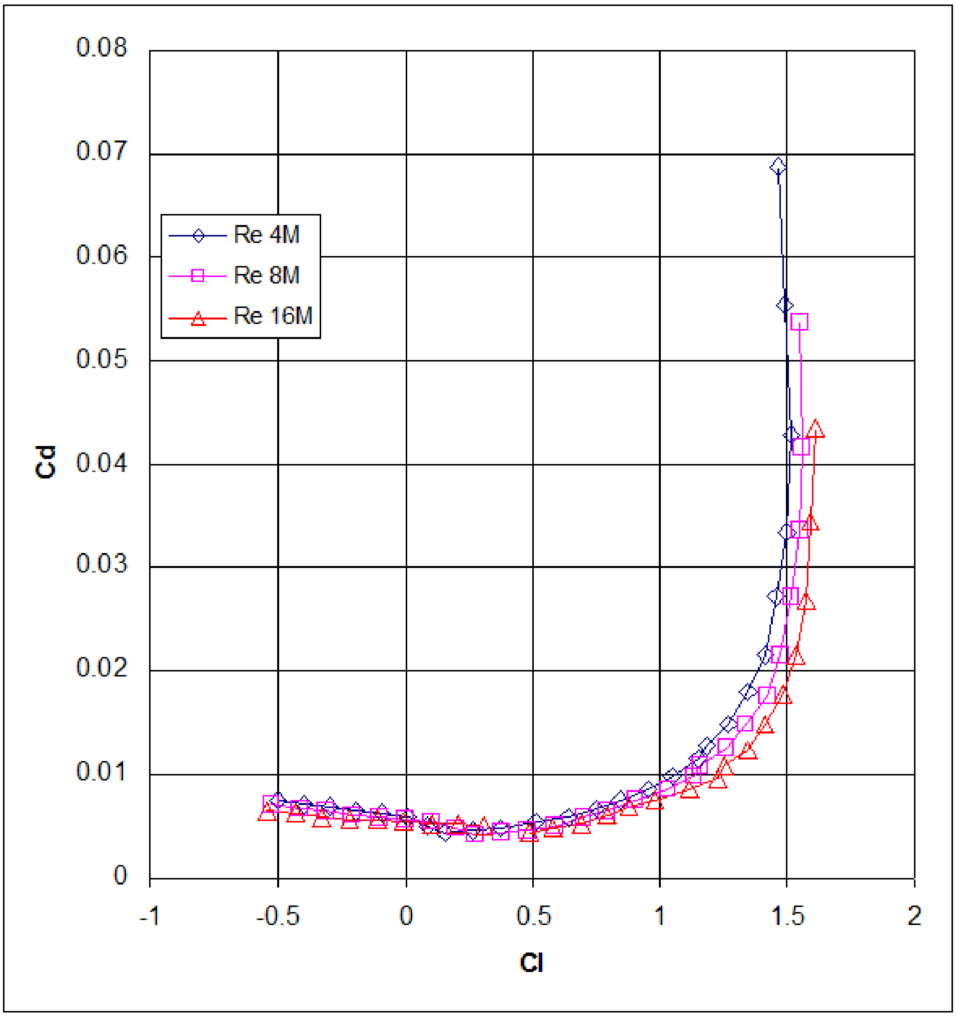

Figure 8. Coefficient of lift vs. coefficient of drag.

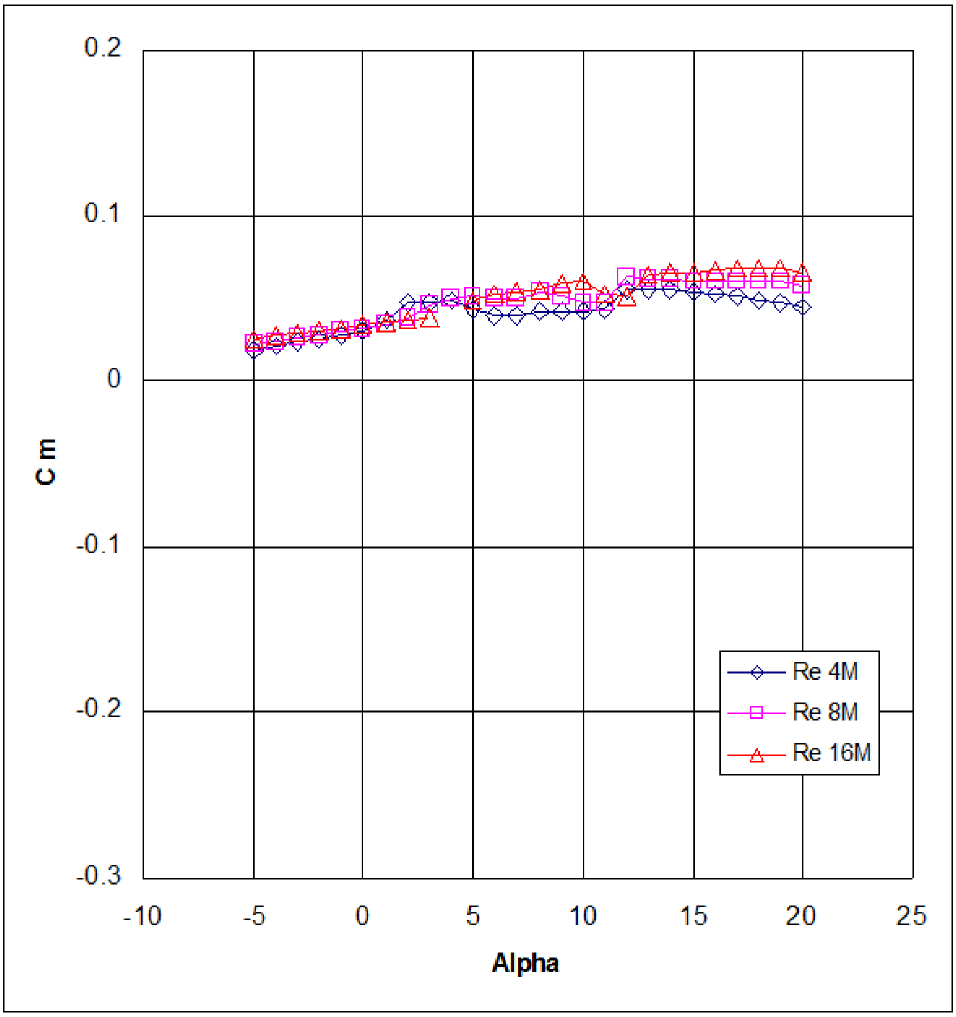

Figure 9. Angle of attack vs. coefficient of moment.

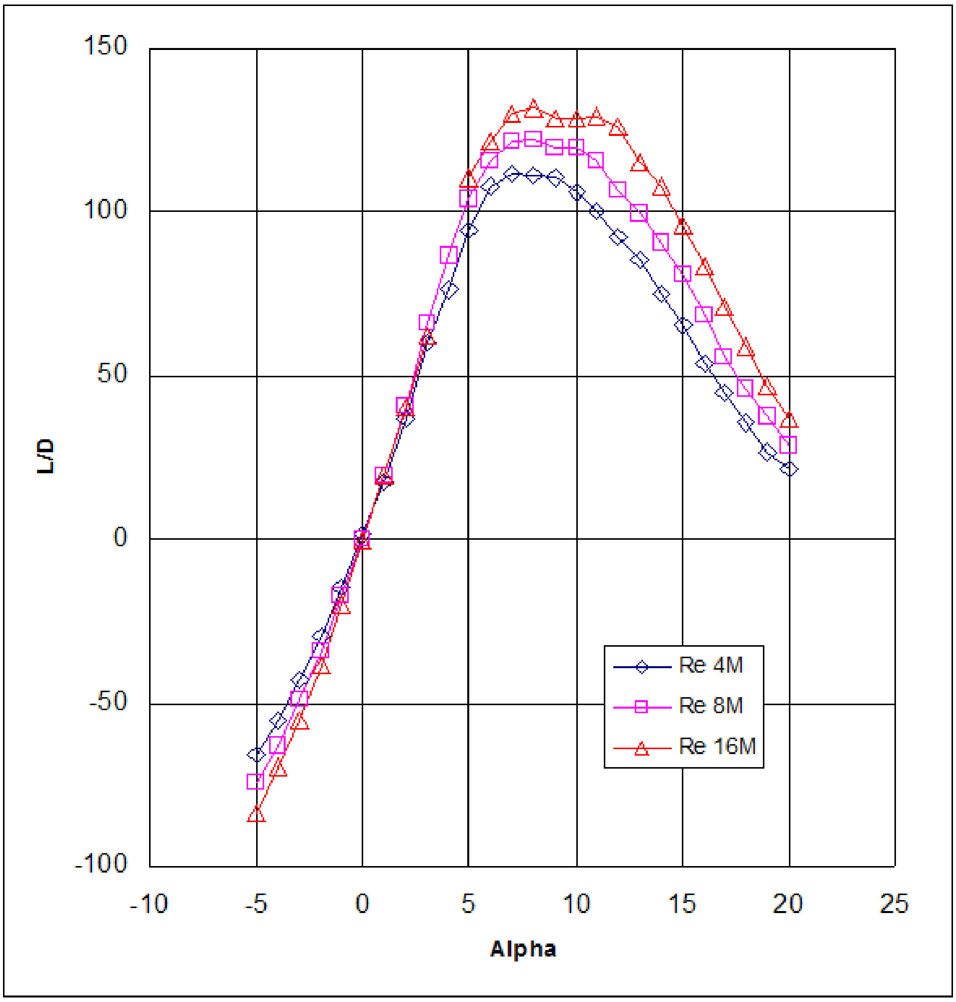

Figure 10. Angle of attack vs. Lift/Drag ratio.

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